FIGURE 2 Showing that the Halting Problem is Unsolvable.

is "halt," then by the definition of $K$ we see that $K(K)$ loops forever, in violation of what $H$ tells us. In both cases, we have a contradiction.

Thus, $H$ cannot always give the correct answers. Consequently, there is no procedure that solves the halting problem.

Exercises

1. List all the steps used by Algorithm 1 to find the maximum of the list 1, 8, 12, 9, 11, 2, 14, 5, 10, 4.

2. Determine which characteristics of an algorithm described in the text (after Algorithm 1) the following procedures have and which they lack.
   a) procedure double(n: positive integer)
      
      while $n > 0$
      
      $n := 2n$
   
   b) procedure divide(n: positive integer)
      
      while $n \geq 0$
      
      $m := \lfloor n/n \rfloor$
      
      $n := n - 1$
   
   c) procedure sum(n: positive integer)
      
      $sum := 0$
      
      while $i < 10$
      
      $sum := sum + i$
   
   d) procedure choose(a, b: integers)
      
      $x := \text{either } a \text{ or } b$

3. Devise an algorithm that finds the sum of all the integers in a list.

4. Describe an algorithm that takes as input a list of $n$ integers and produces as output the largest difference obtained by subtracting an integer in the list from the one following it.

5. Describe an algorithm that takes as input a list of $n$ integers in nondecreasing order and produces the list of all values that occur more than once. (Recall that a list of integers is nondecreasing if each integer in the list is at least as large as the previous integer in the list.)

6. Describe an algorithm that takes as input a list of $n$ integers and finds the number of negative integers in the list.

7. Describe an algorithm that takes as input a list of $n$ integers and finds the location of the last even integer in the list or returns 0 if there are no even integers in the list.

8. Describe all algorithm that takes as input a list of $n$ distinct integers and finds the location of the largest even integer in the list or returns 0 if there are no even integers in the list.

9. A palindrome is a string that reads the same forward and backward. Describe an algorithm for determining whether a string of $n$ characters is a palindrome.

10. Devise an algorithm to compute $x^n$, where $x$ is a real number and $n$ is an integer. [Hint: First give a procedure for computing $x^n$ when $n$ is nonnegative by successive multiplication by $x$, starting with 1. Then extend this procedure, and use the fact that $x^{-n} = 1/x^n$ to compute $x^n$ when $n$ is negative.]

11. Describe an algorithm that interchanges the values of the variables $x$ and $y$, using only assignments. What is the minimum number of assignment statements needed to do this?

12. Describe an algorithm that uses only assignment statements that replaces the triple $(x, y, z)$ with $(y, z, x)$. What is the minimum number of assignment statements needed?

13. List all the steps used to search for 9 in the sequence 1, 3, 4, 5, 6, 8, 9, 11 using
   a) a linear search. 
   b) a binary search.

14. List all the steps used to search for 7 in the sequence given in Exercise 13 for both a linear search and a binary search.

15. Describe an algorithm that inserts an integer $x$ in the appropriate position into the list $a_1, a_2, \ldots, a_n$ of integers that are in increasing order.

16. Describe an algorithm for finding the smallest integer in a finite sequence of natural numbers.

17. Describe an algorithm that locates the first occurrence of the largest element in a finite list of integers, where the integers in the list are not necessarily distinct.

18. Describe an algorithm that locates the last occurrence of the smallest element in a finite list of integers, where the integers in the list are not necessarily distinct.