Question 8: The given equation is the simplest homogenous linear ODE with constant coefficients. This equation taken together with the initial condition \( y(0) = 1 \) constitutes an initial value problem. Let us look for the ODE’s solutions of the form \( y(x) = c_1 e^{\lambda x} \), where \( c_1 \) is a free constant, and \( \lambda \) is a parameter to be determined from the equation.

\[
y' - ay = 0, \quad c_1 \lambda e^{\lambda x} - ac_1 e^{\lambda x} = 0.
\]
\( e^{\lambda x} \neq 0, \ c_1 \neq 0 \) (since \( y(0) \neq 0 \)), \[\Rightarrow \lambda - a = 0, \]
\[\Rightarrow y(x) = c_1 e^{ax}.\]

The obtained \( y(x) \) is the solution to the original homogenous ODE. Now, \( c_1 \) can be determined from the initial condition:

\[y(0) = c_1 e^{a0} = 1,\]
\[\Rightarrow c_1 = 1,\]
\[\Rightarrow y(x) = e^{ax}.\]

This expression for \( y(x) \) is the solution of the initial value problem.

Question 9: The Taylor’s expansion of an \((n + 1)\)-times differentiable function \( f(x) \) in a neighborhood of point \( a \) is as follows:

\[
f(x) = f(a) + \sum_{k=1}^{n} \frac{f^{(k)}(a)}{k!}(x - a)^k + R_{n+1}(x),
\]

where \( R_{n+1}(x) \) is the residual. This formula can be used to approximate \( \cos(x) \) in a neighborhood of 0:

\[
\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + R_5(x).
\]

Question 10: The presented matrix is triangular. Thus, its eigenvalues reside on its main diagonal: \( \lambda_1 = 1, \ \lambda_2 = 6. \)

Question 11: By definition, eigenvalues of a square matrix \( A \in \mathbb{C}^{n \times n} \) are those values of \( \lambda \in \mathbb{C} \) that make matrix \((A - \lambda I)\) singular \((I \in \mathbb{C}^{n \times n} \) is an identity matrix):

\[
\det(A - \lambda I) = 0,
\]
\[\iff \det \begin{bmatrix} 1 & 2 \\ 3 & 4 - \lambda \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0,
\]
\[\iff \det \begin{bmatrix} 1 - \lambda & 2 \\ 3 & 4 - \lambda \end{bmatrix} = 0,
\]
\[\iff (1 - \lambda)(4 - \lambda) - 2 \cdot 3 = 0,
\]
\[\iff \lambda_{1,2} = \frac{5 \pm \sqrt{33}}{2} \approx -0.37, \ 5.37.\]